Variational Principles For Monotone Variational Inequalities

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We consider a parameterized variational inequality (A, Y) in a Banach space E defined on a closed, convex and bounded subset Y of E by a monotone operator A depending on a parameter. We prove that under suitable conditions, there exists an arbitrarily small monotone perturbation of A such that the perturbed variational inequality has a solution which is a continuous function of the parameter, and is near to a given approximate solution. In the nonparametric case this can be considered as a variational principle for variational inequalities, an analogue of the Borwein-Preiss variational principle.

Some applications are given: 1) an analogue of the Nash equilibrium problem, defined by a partially monotone operator, when one of the domain is not a strong compact set. In the case of two functions, this result can be considered as a Sion's type minimax theorem for monotone operators; 2) a variant of the parametric Borwein-Preiss variational principle for Gâteaux differentiable convex functions under relaxed assumptions; 3) a generic result in sense of porosity stating that the most variational inequalities considered here are well posed (which means that the complement of the set of the well posed variational inequalities is σ-porous).

The tool for proving the main result is a useful lemma about existence of continuous ε-solutions of a variational inequality depending on a parameter. It has an independent interest and allows a direct proof of an analogue of Ky Fan's type inequality for monotone operators, introduced here, which leads to a new proof of the Schauder fixed point theorem.

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